edexcel
advancing learning, changing lives

Mark Scheme (Results)
January 2010

GCE

Core Mathematics C3 (6665)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

Ask The Expert can be accessed online at the following link:

http://www.edexcel.com/Aboutus/contact-us/

January 2010

Publications Code US022710

All the material in this publication is copyright
© Edexcel Ltd 2010

January 2010
6665 Core Mathematics C3
Mark Scheme

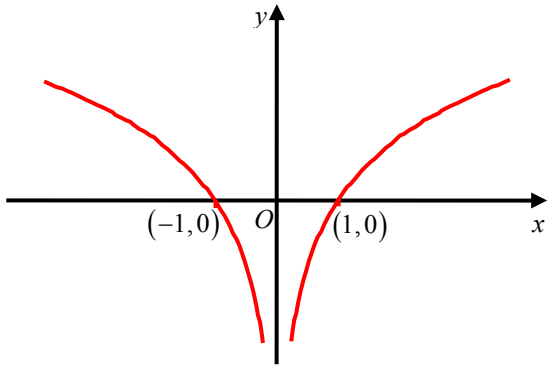
Question Number	Scheme	Marks
Q1	$\frac{x+1}{3x^2-3}-\frac{1}{3x+1}$	
	$=\frac{x+1}{3(x^2-1)}-\frac{1}{3x+1}$	
	$=\frac{x+1}{3(x+1)(x-1)}-\frac{1}{3x+1}$ $x^2-1 \rightarrow (x+1)(x-1)$ or $3x^2-3 \rightarrow (x+1)(3x-3)$ or $3x^2-3 \rightarrow (3x+3)(x-1)$ seen or implied anywhere in candidate's working.	**Award below**
	$=\frac{1}{3(x-1)}-\frac{1}{3x+1}$	
	$=\frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$ Attempt to combine.	M1
	or $\frac{3x+1}{3(x-1)(3x+1)}-\frac{3(x-1)}{3(x-1)(3x+1)}$ Correct result.	A1
	Decide to award M1 here!!	**M1**
	$=\frac{4}{3(x-1)(3x+1)}$ Either $\frac{4}{3(x-1)(3x+1)}$ or $\frac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\frac{4}{(3x-3)(3x+1)}$ or $\frac{4}{9x^2-6x-3}$	A1 **aef**
		[4]

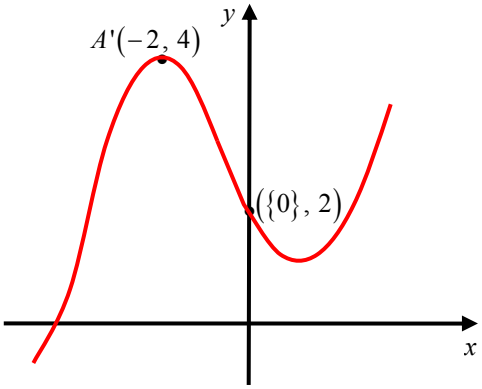
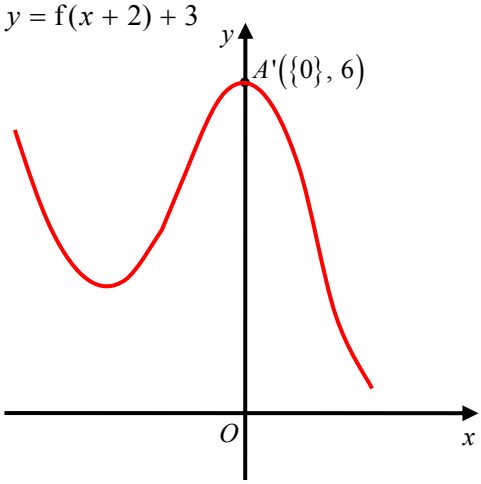
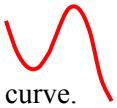
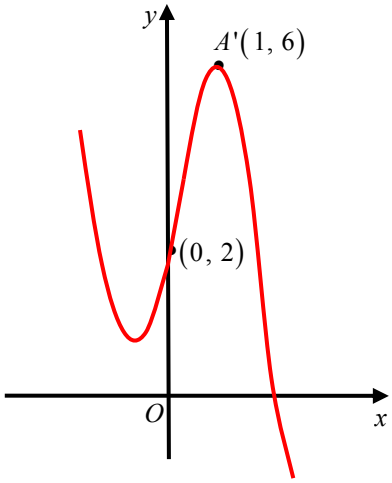
Question Number	Scheme	Marks
Q2	$f(x) = x^3 + 2x^2 - 3x - 11$	
(a)	$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$ Sets $f(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \dfrac{3x+11}{x+2}$ $\Rightarrow x = \sqrt{\left(\dfrac{3x+11}{x+2}\right)}$ then rearranges to give the quoted result on the question paper.	A1 **AG** (2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\dfrac{3x_n+11}{x_n+2}\right)}$, $x_1 = 0$	
	$x_2 = \sqrt{\left(\dfrac{3(0)+11}{(0)+2}\right)}$ An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$ $x_4 = 2.058748112...$ Both x_2 = awrt 2.345 and x_3 = awrt 2.037 x_4 = awrt 2.059	A1 A1 (3)
(c)	Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$	
	$f(2.0565) = -0.013781637...$ $f(2.0575) = 0.0041401094...$ Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp) Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter any one value awrt 1 sf both values correct awrt 1sf, sign change and conclusion As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	M1 dM1 A1 (3) **[8]**

Question Number	Scheme		Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x+\alpha)$, $R>0$, $0<x<\frac{\pi}{2}$		
	$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$		
	$R = \sqrt{5^2+3^2}\,; = \sqrt{34}\ \{= 5.83095..\}$	$R^2 = 5^2 + 3^2$ $\sqrt{34}$ or awrt 5.8	M1; A1
	$\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$	$\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$ $\alpha =$ awrt 0.54 or $\alpha =$ awrt 0.17π or $\alpha = \frac{\pi}{\text{awrt } 5.8}$	M1 A1
	Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x+0.5404)$		(4)
(b)	$5\cos x - 3\sin x = 4$		
	$\sqrt{34}\cos(x+0.5404) = 4$		
	$\cos(x+0.5404) = \frac{4}{\sqrt{34}}\ \{= 0.68599...\}$	$\cos(x \pm \text{their } \alpha) = \frac{4}{\text{their } R}$	M1
	$(x+0.5404) = 0.814826916...^c$	For applying $\cos^{-1}\left(\frac{4}{\text{their } R}\right)$	M1
	$x = 0.2744...^c$	awrt 0.27^c	A1
	$(x+0.5404) = 2\pi - 0.814826916...^c\ \{= 5.468358...^c\}$	$2\pi -$ their 0.8148	ddM1
	$x = 4.9279...^c$	awrt 4.93^c	A1
	Hence, $x = \{0.27, 4.93\}$		(5)
			[9]

Part (b): If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

Question Number	Scheme	Marks
Q4 (i)	$y = \dfrac{\ln(x^2+1)}{x}$	
	$u = \ln(x^2+1) \quad \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$ $\ln(x^2+1) \rightarrow \dfrac{\text{something}}{x^2+1}$ $\ln(x^2+1) \rightarrow \dfrac{2x}{x^2+1}$	M1 A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{array}\right\}$	
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$ Applying $\dfrac{xu' - \ln(x^2+1)v'}{x^2}$ correctly. Correct differentiation with correct bracketing but allow recovery.	M1 A1 (4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1)\right\}$ {Ignore subsequent working.}	
(ii)	$x = \tan y$	
	$\dfrac{dx}{dy} = \sec^2 y$ $\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\dfrac{\sin y}{\cos y}$ using either the quotient rule or product rule. $\dfrac{dx}{dy} = \sec^2 y$	M1* A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} \quad \{= \cos^2 y\}$ Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$.	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$ For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y.	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required) For the correct proof, leading on from the previous line of working.	A1 **AG** (5) **[9]**

Question Number	Scheme	Marks
Q5	$y = \ln\lvert x\rvert$  Right-hand branch in quadrants 4 and 1. Correct shape.	B1
	Left-hand branch in quadrants 2 and 3. Correct shape.	B1
	Completely correct sketch and both $(-1,\{0\})$ and $(1,\{0\})$	B1
		(3)
		[3]

Question Number	Scheme	Marks
Q6 (i)	$y = \mathrm{f}(-x) + 1$ 	
	Shape of 	
	and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\}, 2)$ or $A'(-2, 4)$	B1
	Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x + 2) + 3$ 	
	Any translation of the original curve. 	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)
(iii)	$y = 2\mathrm{f}(2x)$ 	
	Shape of 	
	with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
	Either $(\{0\}, 2)$ or $A'(1, 6)$	B1
	Both $(\{0\}, 2)$ and $A'(1, 6)$	B1
		(3)
		[9]

Question Number	Scheme		Marks
Q7 (a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	$\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$ $-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	M1 A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 **AG** (3)
(b)	$y = e^{2x}\sec 3x$		
	$\left\{\begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{array}\right\}$	Seen or implied Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x \tan 3x$ Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x \tan 3x$	M1 A1
	$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' $2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	M1 A1 **isw** (4)
(c)	Turning point $\Rightarrow \dfrac{dy}{dx} = 0$		
	Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$	Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.	M1
	{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$, }		
	giving $\tan 3x = -\frac{2}{3}$	$\tan 3x = \pm k$; $k \neq 0$	M1
	$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600...$	Either awrt -0.196^c or awrt -11.2°	A1
	Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$ $= 0.812093... = 0.812\ (3\text{sf})$	0.812	A1 **cao** (4)
			[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme	Marks
Q8	$\text{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \qquad 0 \le x \le 180^\circ$	
	Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$ Writing down or using $\text{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\text{cosec}^2\theta = \pm 1 \pm \cot^2\theta$.	M1
	$\underline{\cot^2 2x - \cot 2x} = 0 \quad \text{or} \quad \cot^2 2x = \cot 2x$ For either $\underline{\cot^2 2x - \cot 2x}\ \{= 0\}$ or $\cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0 \quad \text{or} \quad \cot 2x = 1$ Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.	dM1
	$\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$ Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$ Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
	Overall, $x = \{22.5, 45, 112.5, 135\}$ ***Both*** $x = 22.5$ and $x = 112.5$ ***Both*** $x = 45$ and $x = 135$	A1 B1
		[7]

If there are any EXTRA solutions inside the range $0 \le x \le 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180^\circ$.

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}$ $\left\{=51.804...\right\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe
	$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}$ $\left\{=0.0874...\right\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x)=e^{2x}+3,\ x\in\mathbb{R}$		
	$y=e^{2x}+3 \Rightarrow y-3=e^{2x}$ $\Rightarrow \ln(y-3)=2x$ $\Rightarrow \frac{1}{2}\ln(y-3)=x$	Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes ln of both sides	M1 M1
	Hence $f^{-1}(x)=\underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y)=\frac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> cao
	$f^{-1}(x)$: Domain: $\underline{x>3}$ or $\underline{(3,\infty)}$	Either $\underline{x>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Domain}>3}$.	B1
			(4)
(b)	$g(x)=\ln(x-1),\ x\in\mathbb{R},\ x>1$		
	$fg(x)=e^{2\ln(x-1)}+3$ $\left\{=(x-1)^2+3\right\}$	An attempt to put function g into function f.	M1
		$e^{2\ln(x-1)}+3$ or $(x-1)^2+3$ or x^2-2x+4.	A1 isw
	$fg(x)$: Range: $\underline{y>3}$ or $\underline{(3,\infty)}$	Either $\underline{y>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Range}>3}$ or $\underline{fg(x)>3}$.	B1
			(3)
			[15]

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code US022710 January 2010

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH